ON SYSTEMATIC ERRORS IN TIME ESTIMATION.

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It was found in a statistical study on the estimation of time-intervals[1] of different length and of different filling by a great number of subjects that the numerals (0, 1, ... 9) did not occur with the same frequency at the last place. The numerals zero and five occurred with the greatest frequency and the numbers next to zero and five (1 and 9, 4 and 6) occurred with the smallest frequency. Since there was in the statistical data no indication for a further analysis of this fact, it seemed necessary to go back to the study of this phenomenon in individual psychology. An investigation was about to be undertaken when a paper of Mr. O. Meissner[2] was published, which discussed the estimation of short time intervals in terms of fractions (tenths) of a second. These results show striking similarities with ours but they show also some differences and considerable light is thrown on the nature of the mental process of estimating time by comparing both results. I owe to the kindness of Mr. Meissner some important information and valuable help which assisted me greatly in this work.

Mr. Meissner's results refer to the estimation of short time intervals by trained subjects. The results are taken from the observations of transits of stars made by three observers, who are called W, K, and N. N made 12,285 observations; K and N made 8,505 and 16,215 observations respectively. The series of observations was extended over several years and with two of the observers intermissions of considerable length occurred. At the beginning of the series N was 35 years, and W and K were between 20 and 22 years. N was a practiced observer; the practice of W and K was not great but they had had some practice. In so great a number of observations one would suppose that every tenth should occur in the judgments approximately with the same frequency, *i. e.*, in 10% of all the

[1] In a joint publication of the author with Mr. R. M. Yerkes, Time Estimation in its Relation to Sex, Age, and the Physiological Rhythms, 1906, Harvard Psychological Studies, Vol II, pp. 405-430.

[2] Otto Meissner: Ueber systematische Fehler bei Zeit und Raum Schaetzungen, Astronomische Nachrichten, Aug., 1906, Vol. 172, No. 4113. A short report may be found in Sitzungsberichte der Berliner Mathematischen Gesellschaft, 25, April, 1906, pp. 70-72 (Archiv der Mathematik und Physik, Vol. X, Nos. 3 and 4).

cases. The results, however, show that certain numerals are always favored, *i. e.*, they occur in a higher frequency than 10%. This predilection is subjected to variations, which may be considerable and which are apparently quite irregular. This preference for certain numerals constitutes a systematic error which one cannot obviate by combining the results, because only accidental errors can be eliminated in this way. These systematic errors varied much with W, in whose judgments the frequency of zero in the last part of the series was one-half of that in the first part, but they remained fairly constant with N. The fact that N was a practiced observer and that W had had not much practice suggests the view that these variations depend on the practice of the observer.[1]

TABLE I.

Frequency of the Numerals (0 to 9)—W 12,285 Observations, K 8,505 Observations, N 16,215 Observations.

	W	K	N	Average.
0	9.04	21.59	19.23	16.62
1	9.45	3.89	5.24	6.19
2	12.32	4.72	12.49	9.84
3	11.15	9.35	12.52	11.02
4	11.97	15.10	10.91	12.66
5	6.35	11.17	8.09	8.54
6	8.28	10.39	4.45	7.71
7	11.63	9.00	5.19	8.61
8	11.37	7.04	14.37	10.93
9	8.45	7.75	7.52	7.91

The frequencies of all the numerals are given in Table I. The first three columns under the headings W, K and N, give the individual results of these observers and in the last column these results are combined into an arithmetical mean. In the individual results the great personal differences are obvious at once; the frequency of 2, for instance, varies from 2.49% to 12.32%. The general trend of these results becomes clearer by an examination of their average. They show (1), that the small numbers (0 to 4) occur more frequently than the high numbers, the sum of the percentages being for the low numbers 56.33, and for the high numbers 43.70. The sum of these percentages is not exactly 100 owing to an error of computation. (2). The numbers do not occur with equal frequency, zero having the highest frequency, 3 and 8 occurring next with a small difference which might be accounted for by an inaccuracy of the numerical determination. The smallest frequencies are those of 1, 6 and 9 in the order given.

[1] See Meissner: l. c. Tables I, II and III.

For a comparison of Mr. Meissner's results with ours Table III (Males) of the article quoted above comes into consideration. These numbers are not directly comparable with Mr. Meissner's results because the estimation of long time intervals is not only under the influence of the preference for certain numerals but also under the influence of the conventional minute standard. The numbers favored by the latter influence are the simple fractions of a minute (15″, 30″, 45″ and 60″) and their multiples. The final digit of such numbers is 0 or 5, the frequency of which is increased by this influence, which must be eliminated for the comparison because the results of the estimation of short time intervals are free from it. This can be done in this way. Among all the judgments given there are 1,666, the final digit of which is zero. According to chance one-third of these (555) should be multiples of 30, because among three consecutive multiples of 10 there is one multiple of 30. As a matter of fact there are 732 and by subtracting this surplus of 207 the influence of the minute standard may be eliminated. A similar computation may be made for the numeral 5. On the basis of this computation one may correct the relative frequencies of the other numerals; the results are given in Table II.

TABLE II.

Corrected Frequency of the Numerals 0 to 9.

	Frequency.	Percentage.	True Value.
0	1177	36.724	
1	121	3.775	0.202
2	217	6.771	0.255
3	155	4.836	0.313
4	144	4.493	0.360
5	660	20.593	0.485
6	153	4.774	0.612
7	170	5.304	0.663
8	281	8.767	0.733
9	127	3.963	0.796
	3205	100.000	

A comparison of these results with the numbers of Table III of the previous article will show that this change does not affect the relative frequencies in a material way. The most favored numeral in this case, too, is zero, the next is 5 and then comes 8. The numerals 1, 9, 4 and 6 have the smallest frequency in the order given. These results agree with Mr. Meissner's observations by giving the greatest frequency to 0 and by showing that 8 is favored and that 1, 9 and 6 are at a disadvantage. They do not agree in regard to the frequency of

5 and 4. The most striking difference is that of the numerical values of these relative frequencies. This difference is due to the overwhelming frequency of 0 and 5 which affects the frequency of the other numerals in such a way that they are throughout smaller than those of the observations of Mr. Meissner. The influence in favor of 0 and 5 is much stronger in our experiments.

This partial verification of these two series of experiments might be interpreted as indicating a common factor in both series of observations, the influence of which was partially counterbalanced by some other circumstances. The common factor may be called the preference for certain numerals and the nature of the other influence can be studied by means of the following considerations.

Among the results of W, there are 9.04% cases in which he observed the interval 0 and 9.45% cases in which he observed the interval 0.1." This means that all these intervals which differ by not more than 0.0452" from zero—*i. e.*, the interval from—0.9548" to +0.0452"—are judged to be zero. Correspondingly the end of the interval which he considers to be 0.1" has the distance from zero 0.045 + ½.0.94 = 0.045 + 0.047 = 0.092". This means that all the intervals from 0.046" to 0.137" are judged to be 0.1", and 0.092", the average may be regarded as the interval corresponding to W's estimation of 0.1". By continuing this operation one may find for all the other numerals the corresponding intervals which are called the true values of the estimation. They are given in Table III for Mr. Meissner's observations[1] and in Table II under the heading

TABLE III.

	W	K	N
0.0	0.000	0.000	0.000
0.1	0.092	0.127	0.122
0.2	0.202	0.171	0.211
0.3	0.319	0.241	0.337
0.4	0.434	0.364	0.404
0.5	0.527	0.495	0.548
0.6	0.599	0.603	0.611
0.7	0.699	0.700	0.659
0.8	0.814	0.780	0.757
0.9	0.913	0.849	0.867

"True Value" for our experiments. Table III shows that the estimated intervals coincide most closely with the actual intervals for 0.6" and 0.7", and that W has the tendency to overestimate and K to underestimate. The judgments of N show

[1]See Meissner's l. c. p. 141, 142.

a constant overestimation for the intervals from 0.1″ to 0.6″ and from this point the overestimation turns into an underestimation, the amount of error being smallest and the accuracy being greatest for 0.6.″ The judgments of W and K bear out the same relation: the exactitude is greatest for 0.6″ and 0.7.″

These figures agree very well with the results of previous psychological investigations. Small time intervals are, as a rule, overestimated and larger time intervals are underestimated. There lies between the large and the small intervals a point of indifference at which the estimated length of the intervals correspond most accurately to the real length. This point lies, according to different observers, between 0.5″ and 0.7″ and uniting these results into a mean one may take 0.6″ for this point.[1] These conditions are exactly verified in the observer N who had many years practice in these observations. The two other observers show slight differences. W overestimates small intervals and is most exact in his estimations of intervals of 0.6″ and 0.7″ but he overestimates also the intervals of 0.8″ and 0.9″. K underestimates long and short intervals but also his estimation is most correct for 0.6″ and 0.7″. These facts account for the great frequency of the small numerals.

The great frequency of zero, too, can be explained from the conditions of the experiments. The judgment zero will be given if the star crosses the line in the moment of a beat of the clock. From the experiments with the "Complicationsuhr" we know that for the most part the sound is located at a point which is distinguished from the other points by some marks. Thus it occurs but seldom that the sound is located between the dashes of the clock.[2] The lines in the telescope play the same rôle as the marks on the dial of the clock and, therefore, the beat of the clock is heard more frequently at the moment of the star crossing the line. This accounts not only for the excessive frequency of zero but also for the low frequency of 1 and 9, because if in a series of equal small time intervals one interval is marked in such a way as to induce an overestimation of it, then the overestimation is made at the expense of the adjoining intervals which are underestimated.

It is obvious that a similar computation of the "true values" for our results cannot have the same signification. The results, indeed, show that the estimation comes nearest to the

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[1] Wundt, Physiologische Psychologie, 5. ed. Vol. III, p. 47. Wundt refers to the investigations of Kollert, Estel and Meumann; a thorough discussion of their results is to be found in I. Quandt, Das Problem des Zeitbewusstseins, Litteraturbericht, Arch. f. d. ges. Psychologie, 1906, Vol. VIII, pp. 143-189.

[2] Compare Wundt: Physiologie Psychologie, 5 ed. Vol. III, p. 69.

true value for 0.6″, but the change from overestimation to underestimation is quite irregular. The intervals from 0.1″ to 0.3″ are overestimated, then this overestimation changes into underestimation for 0.4″ and 0.5″. The interval of 0.6″ is overestimated again but the other intervals from 0.7″ to 0.9″ are underestimated. This warrants our statement that the factor, the influence of which on the judgment of short intervals was just proved, is not at work in the estimation of large intervals. The explanation of the very high frequency of zero and five in the estimation of long intervals is to be found merely in the general inaccuracy of the estimation which is expressed by the preference for round numbers.[1] The low frequency of the numbers which are next to the round numbers may be explained by the subject feeling the inaccuracy of the judgment which does not warrant a deviation from a round number by so small a margin. The view that the low frequency of these numerals is due to underestimation of the seconds just preceding or following every fifth and tenth second has but little psychological probability. It is contradicted also by introspection which shows that the judgment in this case is not based on the perception of a succession of small intervals among which, perhaps, every fifth or tenth is accentuated, but the estimation refers to the vague notion of intervals of approximately the same duration. It does not seem possible to give at present a reason for the differences in the frequencies of the other number (2, 3, 8 and 9), among which the even numbers occur with a higher frequency than the uneven. An explanation is not offered by saying that the different frequencies of the numerals are due to a preference for certain numerals, because, without introspective or objective evidence for such a predilection, the only possibility of giving an account for this preference is to refer to the different frequencies. "Preference for certain numerals" and "different frequencies of certain numerals" are only different names for the same thing and the first term is misleading in so far as it might lead one to suppose that there exists a certain mental process which in every case is the cause of the differences in the frequencies of the numerals. There is positive evidence that such a process does not exist. If there existed such a mysterious process then it could not happen that the same numeral occurs with a frequency which is subjected to variations which are so large as those of the frequency of 4 and 5 in our experiments and in those of Mr. Meissner's series. It seems reasonable to take the view that the terms "preference for certain numerals" is a

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[1] Other reasons for this statement may be found in the article on time estimation, pp. 411-418.

short designation for an unanalyzed group of conditions which is the cause of the differences in the frequencies of the numerals; these groups of conditions may be widely, or perhaps even absolutely, different in two series of experiments.

The comparison of these two series of observations shows that there are certain common results (high frequency of certain numerals, low frequency of the numerals adjoining to them). The high frequencies are apparently due to different mental conditions, but the low frequencies of the adjoining numerals may be brought under the general rule that the elements of a complex which are next to a favored complex with which they are not associated are at a disadvantage. The chief result is that the so-called preference for certain numerals is a complicated mental process which admits of a further psychological analysis. The analysis of these two series of observations shows that this preference for certain numerals may be very similar in its numerical expression and yet it may be due to entirely different conditions.